Lecture 16 - Bose-Einstein condensation continued

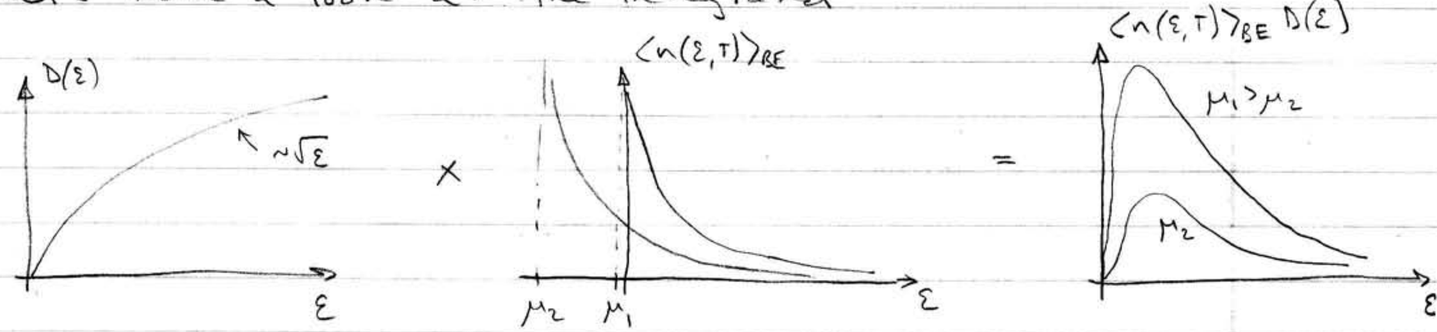
For a box filled with bosons with spin 0

$$D(\varepsilon) = \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon}$$

Note no factor of 2 because there is only one spin state

$$\therefore \quad N_e(T) = \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{\sqrt{\varepsilon}\,d\varepsilon}{e^{\beta(\varepsilon-\mu)}-1} = F(\mu,T)$$

Let's take a look at the integrand



* Notice the function $\langle n(\varepsilon,T)\rangle_{BE}\,D(\varepsilon) = 0$ at $\varepsilon = 0$, so it does not include the gd. state

Area under the curve is the integral, so $F(\mu,T)\uparrow$ as $\mu\uparrow$ from a negative value to 0

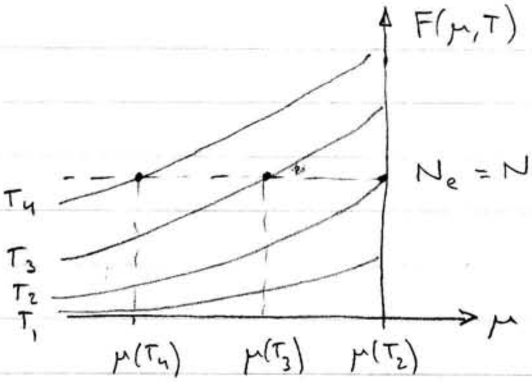
How does $F(\mu,T)$ depend on T?

Define $x \equiv \beta\varepsilon$

$$\text{so} \quad F(\mu,T) = \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}(k_B T)^{3/2}\int_0^\infty \frac{x^{1/2}\,dx}{e^{x-\beta\mu}-1}$$

So, overall, $F(\mu,T)\uparrow$ as $T\uparrow$

So graphically $N_e(T) = F(\mu, T)$ is



$T_4 > T_3 > T_2 > T_1$

Solve for $\mu(T)$ from intersection of $F(\mu, T)$ with horizontal line at N_e

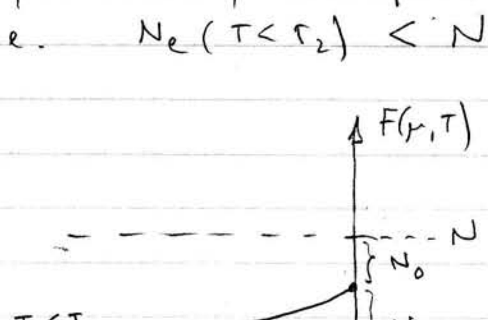
Let's imagine we start at a sufficiently high T, where boson gas behaves classically, and cool down

At T_4, μ is a large negative number $\left(\mu = -k_B T \ln\left(\frac{n_Q}{n}\right) \right)$ so # bosons in gd. state will be negligibly small

$$N_0 = \frac{1}{e^{\beta(\varepsilon_0 - \mu)} - 1} \approx \frac{1}{e^{-\beta\mu} - 1} \approx e^{-\beta|\mu|} \ll 1$$

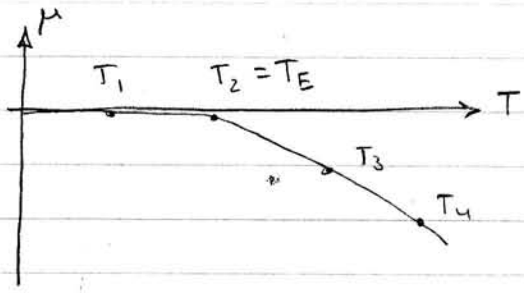
So all bosons are in excited states $N_e(T_4) = N_{tot} = N$

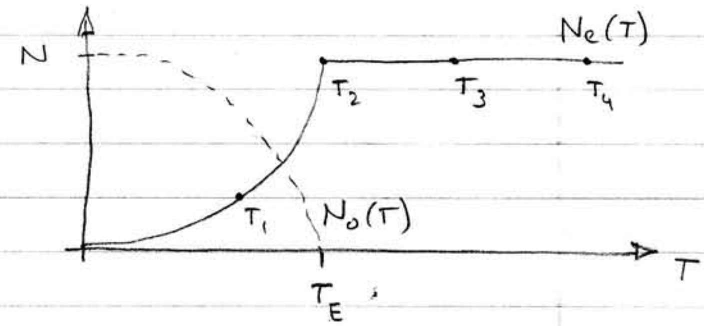
Same true for T_3, down to T_2. As we cool μ approaches 0
What happens for $T < T_2$? $F(\mu, T<T_2) < N$ so there is no way to satisfy the equation, unless N_e also drops,
i.e. $N_e(T<T_2) < N$ so N_0 must increase dramatically



For $T_1 < T_2$, N_0 is a sizable fraction of N. Macroscopic # of bosons occupy ground state
<u>Bose-Einstein</u> <u>condensation</u>

Let's plot what happens to $\mu(T)$ and $N_e(T)$





Temperature at which this starts to happen is T_E $(= T_2)$
The Einstein condensation temperature

Graphs tell us qualitatively what happens. Now let's calculate it

Note that at $T = T_E$ $\quad \mu \approx 0$* (actually $\mu = \varepsilon_0 - \frac{kT}{N} \approx 0$)
and $N_e = N$ still so

$$N_e(T_E) = F(\mu \approx 0, T_E)$$

$$= \underbrace{\frac{V}{4\pi^2}\left(\frac{2m k_B T_E}{\hbar^2}\right)^{3/2}}_{\frac{V}{4\pi^2}\cdot(4\pi)^{3/2}\left(\frac{2\pi m k_B T_E}{h^2}\right)^{3/2} = \frac{2}{\sqrt{\pi}} V n_Q(T_E)} \underbrace{\int_0^\infty \frac{x^{1/2}\,dx}{e^x - 1}}$$

$$\int_0^\infty \frac{x^{1/2} e^{-x}}{1 - e^{-x}}\,dx = \sum_{s=1}^{\infty} \int_0^\infty x^{1/2} e^{-sx}\,dx$$

$$= \sum_{s=1}^{\infty} \frac{\Gamma(3/2)}{s^{3/2}} = \zeta\left(\tfrac{3}{2}\right)\Gamma\left(\tfrac{3}{2}\right)$$

$$= 2.612 \cdot \frac{\sqrt{\pi}}{2}$$

$$\therefore\ N_e(T_E) = N = 2.612\, n_Q(T_E)\, V$$

* $\mu \approx 0$ is a fine approximation since $\varepsilon_1 \gg \mu$

So: $k_B T_E = \left(\frac{n}{2.612}\right)^{2/3} \cdot \frac{h^2}{2\pi m}$ or $n = 2.612\, n_Q(T_E)$
(Note $n \sim n_Q$!)

Below T_E μ stays "locked" at $\mu \approx 0$, so the same formula holds for $T \leq T_E$

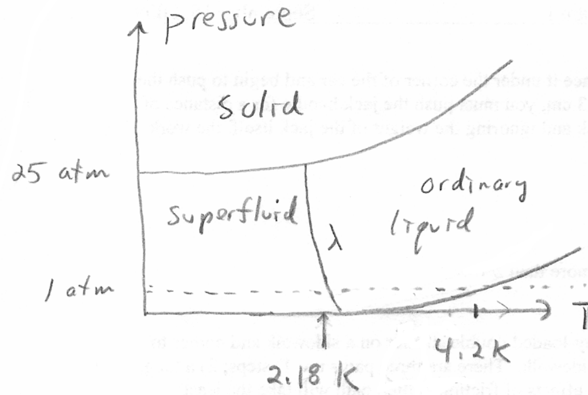
$$N_e(T) = 2.612\, n_Q(T) V \qquad \text{for } T \leq T_E$$

Divide this by $N = N_e(T_E) = 2.612\, n_Q(T_E) V$
Fraction of bosons in excited states is:

$$\frac{N_e(T)}{N} = \frac{n_Q(T)}{n_Q(T_E)} = \left(\frac{T}{T_E}\right)^{3/2} \qquad \text{for } T \leq T_E$$

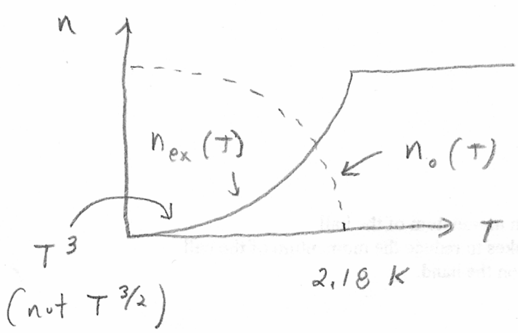
Since $N_0 = N - N_e$:

$$\boxed{N_0 = N\left(1 - \left(\frac{T}{T_E}\right)^{3/2}\right)}$$



For $p < 25$ atm ^4He is a liquid down to $T = 0$. But once it reaches the "λ" line, it becomes a two-component fluid.

One component is somewhat like the condensate and flows without viscosity - called a "superfluid". The other component is more like an ordinary fluid.



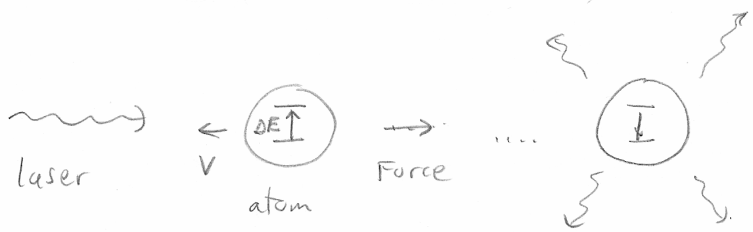
Certainly Bose-Einstein condensation has something to do with it, but interactions are too strong to calculate its properties precisely.

BEC in dilute atomic gases ^{87}Rb, ^7Li, ^{23}Na

^{87}Rb (37p, 37e, 50n) Even # of fermions so the atom acts like a boson. Idea is to use dilute gases so the BEC can form before they form molecules or clusters.

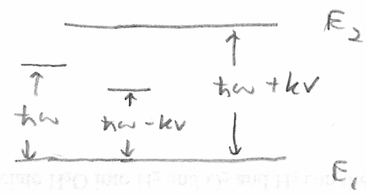
$n \sim 10^{14}/cm^3$, $T_{BEC} \sim 10^{-9}$ Kelvin !

How to cool the atoms

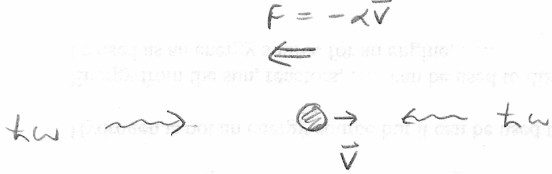


Laser sends in photons that the moving atom absorbs. These are re-emitted with some probability in all directions. So net result is that after each collision, re-emission process the atoms are slowed down.

Optical Molasses: Consider atom with 2 energy levels separated by $\Delta E = E_2 - E_1$. Now send in laser beams with $\hbar\omega < \Delta E$



$$\vec{F} = -\alpha \vec{v}$$



Atom moving toward beam at speed v sees light Doppler shifted up by $+kv$ so it is closer to ΔE and more likely to be absorbed, and slowed down. It sees the beam "following" it at $\hbar\omega - kv$ so it is less effective. Net result is a force $\vec{F} = -\alpha\vec{v}$ no matter what direction it moves. Just like viscous damping. "Optical molasses" (S. Chu - Nobel)

Magnetic Trapping

Optical molasses can cool atoms down to $\sim 50-100\ \mu K$. This isn't cold enough. But it is possible to use the magnetic properties of the atom for further cooling and trapping.

Total angular momentum of the atom $\vec{F} = \vec{J} + \vec{I}$

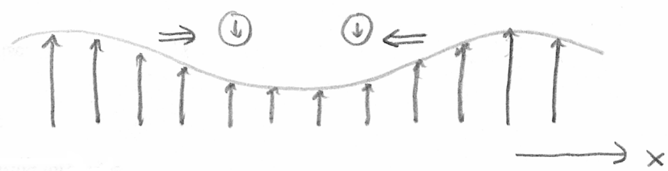
electrons ↗ ↑ nucleus

In a $\vec{B}$-field this gives rise to hyperfine levels F, $m_F = -F, -F+1, \ldots, +F$. Atom therefore has a magnetic moment $\vec{\mu} = g_F \mu_B \vec{F}$.

$$\text{Energy} = -\vec{\mu}\cdot\vec{B} = -g_F \mu_B m_F B$$

So depending upon the m_F state, atom can be attracted or repelled by an inhomogeneous $\vec{B}$ field.

Say $\vec{B} = B_z(x)\,\hat{z}$



$$F_x = -\frac{\partial U}{\partial x} = g_F \mu_B m_F \frac{dB_z}{dx}$$

$m_F = -1$ atoms would be sucked in.

Typically, coils are designed to produce a magnetic trap with effective potential energy

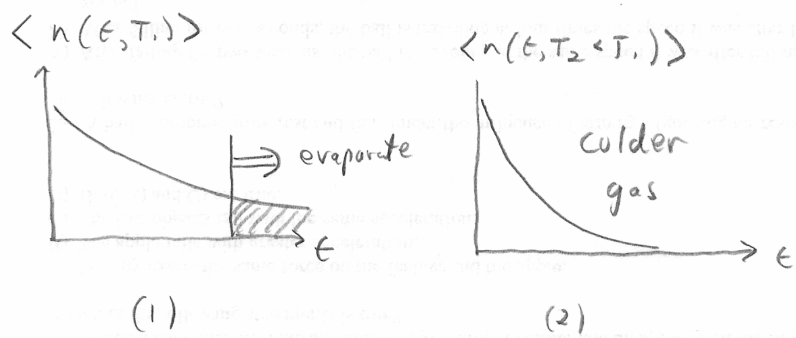
$$U = a z^2 + b(x^2+y^2)$$

This a harmonic oscillator potential so the states have energies

$$E = \hbar\omega_a\left(n_z+\tfrac{1}{2}\right) + \hbar\omega_b\left(n_x+\tfrac{1}{2}\right) + \hbar\omega_b\left(n_y+\tfrac{1}{2}\right)$$

Evaporative Cooling

Final stage of cooling involves applying radio-frequency $\vec{B}$ fields to induce transitions between different m_F levels. By promoting atoms into $m_F = +1$ state the will be untrapped an evaporated. This cools the gas to nano-kelvin temperatures.



Successive evaporations cool enough for BEC.